



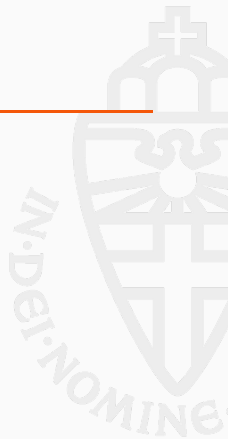
agdarsec -- Total Parser Combinators

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Brouwer Seminar

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- Host language's tooling & libraries
- Help from the coverage & type checkers
- Higher-Order Parser (+ fixpoints)



```
type Parser a =  
    String          -- input string  
    -> [(a, String)] -- possible values + leftover  
  
parse :: Parser a -> String -> Maybe a  
parse p s = case filter (null . snd) (p s) of  
    [(a, [])] -> Just a  
    _         -> Nothing
```

```
anyChar :: Parser Char
```

```
anyChar [] = []
```

```
anyChar (c : s) = [(c, s)]
```

```
guard :: (a -> Bool) -> Parser a -> Parser a
```

```
guard f p s = filter (f . fst) (p s)
```

```
digit :: Parser Char
```

```
digit = guard (`elem` "0123456789") anyChar
```

Our First Combinators

```
anyChar :: Parser Char
```

```
anyChar [] = []
```

```
anyChar (c : s) = [(c, s)]
```

```
guard' :: (a -> Maybe b) -> Parser a -> Parser b
```

```
guard' f p s = catMaybes $ fmap check (p s) where
```

```
  check (a, s) = fmap (,s) (f a)
```

```
digit' :: Parser Int
```

```
digit' = guard' (readMaybe . (:[])) anyChar
```

```
instance Functor Parser where (...)  
instance Applicative Parser where (...)  
instance Monad Parser where (...)  
  
instance Alternative Parser where  
  empty :: Parser a  
  empty s = []  
  
(<|>) :: Parser a -> Parser a -> Parser a  
(p <|> q) s = p s ++ q s
```



```
some :: Parser a -> Parser [a]
```

```
some p = (:) <$> p <*> many p
```

```
many :: Parser a -> Parser [a]
```

```
many p = some p <|> pure []
```



```
data Expr = Literal Int | Plus Expr Expr
```

```
char :: Char -> Parser Char
```

```
char c = guard (c ==) anyChar
```

```
int :: Parser Int
```

```
int = convert <$> some digit' where  
  convert ds = (...)
```

```
expr :: Parser Expr
```

```
expr = Literal <$> int
```

```
  <|> Plus <$> expr <*> char '+' <*> expr
```




```
data Expr = Literal Int | Plus Expr Expr
```

```
char :: Char -> Parser Char
```

```
char c = guard (c ==) anyChar
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```
int :: Parser Int
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```
int = convert <$> some digit' where
```

```
  convert ds = (...)
```

```
expr :: Parser Expr
```

```
expr = Literal <$> int
```

```
  <|> Plus <$> expr <^ char '+' <^> expr
```

```
data Expr = Literal Int | Plus Expr Expr
```

```
char :: Char -> Parser Char
```

```
char c = guard (c ==) anyChar
```

```
int :: Parser Int
```

```
int = convert <$> some digit' where
```

```
  convert ds = (...)
```

```
expr' :: Parser Expr
```

```
expr' = base <|> Plus <$> base <*> char '+' <*> expr'
```

```
  where base = Literal <$> int
```

```
        <|> char '(' *> expr' <*> char ')'
```



```
|_|≡_ : {A : Set} → List A → ℕ → Set
```

```
| []      |≡ zero  = T
```

```
| x :: xs |≡ suc n = | xs |≡ n
```

```
| []      |≡ suc n = ⊥
```

```
| x :: xs |≡ zero  = ⊥
```

```
record |List_|≡_ (A : Set) (n : ℕ) : Set where
```

```
  constructor mkSizedList
```

```
  field list      : List A
```

```
    .proof : | list |≡ n
```



```
record Success (Tok : Set)
  (A : Set) (n : ℕ) : Set where
constructor _^_,_
field
  value      : A
  {size}     : ℕ
  .small     : size < n
  leftovers  : |List Tok |≡ size
```



```
record Parser (Tok : Set) (M : Set → Set)
  (A : Set) (n : N) : Set where
  constructor mkParser
  field runParser : ∀ {m} → .(m ≤ n) →
    |List Tok | ≡ m → M (Success Tok A m)
```

$_ \rightarrow _ : (A\ B : I \rightarrow \text{Set}) \rightarrow (I \rightarrow \text{Set})$

$(A \rightarrow B)\ n = A\ n \rightarrow B\ n$

$_ \oplus _ : (A\ B : I \rightarrow \text{Set}) \rightarrow (I \rightarrow \text{Set})$

$(A \oplus B)\ n = A\ n \uplus B\ n$

$_ \otimes _ : (A\ B : I \rightarrow \text{Set}) \rightarrow (I \rightarrow \text{Set})$

$(A \otimes B)\ n = A\ n \times B\ n$

$[_] : (A : I \rightarrow \text{Set}) \rightarrow \text{Set}$

$[A] = \forall \{n\} \rightarrow A\ n$



```
record □_ (A : ℕ → Set) (n : ℕ) : Set where
  constructor mkBox
  field call : ∀ {m} → .(m < n) → A m
```

```
map : [ A → B ] → [ □ A → □ B ]
```

```
app : [ □ (A → B) → (□ A → □ B) ]
```

```
duplicate : [ □ A → □ □ A ]
```

```
extract : [ □ A ] → [ A ]
```

```
fix : ∀ A → [ □ A → A ] → [ A ]
```

```
loeb : [ □ (□ A → A) → □ A ]
```



Precise Types for Parser Combinators

`anyTok` : [Parser Tok M Tok]

`guardM` : (A → Maybe B) →
[Parser Tok M A → Parser Tok M B]

`_ \langle $ \rangle _` : (A → B) →
[Parser Tok M A → Parser Tok M B]

`return` : [Parser Tok M A → \square Parser Tok M A]

`_ \langle * \rangle _` : [Parser Tok M (A → B) → Parser Tok M A
→ Parser Tok M B]

`_ \langle | \rangle _` : [Parser Tok M A → Parser Tok M A
→ Parser Tok M A]


```
_&?>>=_ : [ Parser Tok M A  
            → (const A → □ Parser Tok M B)  
            → Parser Tok M (A × Maybe B) ]
```



Reminder: Non Total

```
data Expr = Literal Int | Plus Expr Expr
```

```
char :: Char -> Parser Char
```

```
char c = guard (c ==) anyChar
```

```
int :: Parser Int
```

```
int = convert <$> some digit' where
```

```
  convert ds = (...)
```

```
expr :: Parser Expr
```

```
expr = Literal <$> int
```

```
  <|> Plus <$> expr <^ char '+' <^> expr
```

Reminder: Non Total

```
data Expr = Literal Int | Plus Expr Expr
```

```
char :: Char -> Parser Char
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```
char c = guard (c ==) anyChar
```

```
int :: Parser Int
```

```
int = convert <$> some digit' where
```

```
  convert ds = (...)
```

```
expr :: Parser Expr
```

```
expr = fix $ \ rec -> Literal <$> int
```

```
  <|> Plus <$> rec <*> char '+' <*> rec
```

Combinator for Guarded Recursion

```
expr :: Parser Expr
expr = fix $ \ rec -> Literal <$> int
  <|> Plus <$> rec <*> char '+' <*> rec
```

`fix` : $\forall A \rightarrow [\square A \rightarrow A] \rightarrow [A]$

`<$>` : $(A \rightarrow B) \rightarrow$
[Parser Tok M A \rightarrow Parser Tok M B]

`<|>` : [Parser Tok M A \rightarrow Parser Tok M A
 \rightarrow Parser Tok M A]

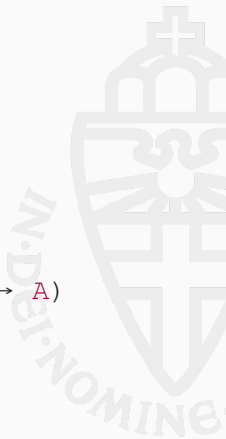
`<&` : [Parser Tok M A \rightarrow \square Parser Tok M B
 \rightarrow Parser Tok M A]

`<&>` : [Parser Tok M A \rightarrow \square Parser Tok M B
 \rightarrow Parser Tok M (A \times B)]

We can still recover the fixes

```
hchain1 : [ Parser Tok M A
           → □ Parser Tok M (A → B → A)
           → □ Parser Tok M B
           → Parser Tok M A ]
```

```
chain(l/r)1 : [ Parser Tok M A
                → □ Parser Tok M (A → A → A)
                → Parser Tok M A ]
```



And Safely Implement a Parser for Expr

```
data Expr : Set where
  Var      : Char → Expr
  Lit      : N → Expr
  Add Sub Mul Div : Expr → Expr → Expr

expr : [ Parser Char Maybe Expr ]
expr = fix (Parser Char Maybe Expr) $ λ rec →
  let var      = Var <$> alpha
      lit      = Lit <$> decimal
      addop    = Add <$ char '+' <|> Sub <$ char '-'
      mulop    = Mul <$ char '*' <|> Div <$ char '/'
      factor   = parens rec <|> var <|> lit
      term     = chainl1 factor $ return mulop
      expr     = chainl1 term $ return addop
  in expr
```

`https://github.com/gallais/agdarsec`

