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### Introduction

Session types are a channel typing discipline ensuring that the communication patterns of concurrent programs abide by a shared protocol. A meta-theoretical analysis of the typing discipline can then ensure that the communicating processes will have good properties e.g. deadlock-freedom. We show how to use a linear dependently typed programming language to define a direct and type-safe embedding of expressive binary session types.

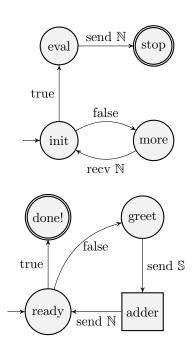
### **Expressive Session Types**

Our session types are closed under nested smallest  $(\mu x. \cdot)$ and largest  $(\nu x. \cdot)$  fixpoints, typed sends  $(!A. \cdot)$  and receives  $(?A. \cdot)$ , the empty protocol (end) and binary branch offerings  $(\cdot \& \cdot)$  and selections  $(\cdot \oplus \cdot)$ .

The session  $\nu i$ .  $((!\mathbb{N}. end) \& (?\mathbb{N}. i))$  types a server for the 'adder' protocol represented as a finite state machine on the right hand side. The server accepts an arbitrarily long sequence of natural numbers (sent by a client repeatedly selecting the *more* =  $(?\mathbb{N}. x)$  branch) before sending back their sum as a single number (when the *eval* =  $(!\mathbb{N}. end)$  branch is selected) and terminating. The encoding of the server's type uses a largest fixpoint because the client is the one responsible for ensuring the process terminates by eventually selecting the *eval* branch.

We can use nested fixpoint to describe more complex protocols. For instance, the session described by  $\nu r.$  (end &(!S.  $\nu i.$  ((!N. r) &(?N. i))))) informally corresponds to the finite state machine shown on the right hand side. It types a server repeatedly offering to handle the adder protocol described above after greeting the user with a string. We replaced adder's end with r to let the protocol loop back to the 'ready' state once the adder is done (in the state machine, that corresponds to merging 'stop' and 'ready' hence the bottom 'send N'-labeled arrow from the 'adder' sub-machine back to 'ready').

# $p, q, \dots ::= x \mid \mu x. p \mid \nu x. p \\ \mid !A. p \mid ?A. p \mid end \\ \mid p \& q \mid p \oplus q$



### Type-safe Implementation

**Pairs of Unityped Channels** We represent a bidirectional channel parametrised by a session type as a pair of unityped unidirectional channels. The key idea behind this safe implementation is to collect all the received (respectively sent) types used in a protocol and to manufacture a big sum type in which we can inject all of the received (respectively sent) values. Because

the received types of a protocol are equal to the sent types of its dual (and vice-versa), we can prove that the directed channels of two communicating parties are indeed type-aligned. The definition of this sum type is a type safe realisation of Kiselyov and Sabry's open union type [5] using the encoding of scope-safe De Bruijn indices [4] introduced by Brady in Idris 2's core [3]. We include its definition below: given a list ts of values of type a and a decoding function elt turning such values into types, (UnionT elt ts) defines a union type. Its one constructor named Element takes a natural number k functioning as a tag, a proof of type (AtIndex t ts k) that the value t is present at index k in ts and injects a value of type (elt t) into the union. By picking elt to be the identity function, we recover the usual notion of union of types.

```
data UnionT : (elt : a -> Type) -> (ts : List a) -> Type where
Element : (k : Nat) -> (0 _ : AtIndex t ts k) -> elt t -> UnionT elt ts
```

Note that the proof of type (AtIndex t ts k) is marked as erased through the use of the 0 quantity which means this datatype compiles down to a pair of a GMP-style Integer (the optimised runtime representation of Nat) and a value of the appropriate type. This gives us an encoding that has constant time injections and partial projections (via an equality check on the Nat tag) in and out of the big sum type.

**Dealing with Changing Sessions** The session type, by design, changes after each communication but the big sum types used when communicating across the unityped channels needs to stay the same. Our solution is to record an offset remembering where we currently are in the protocol and thus allowing us to keep injecting values into the initial shared sum types. Our offsets are proven correct using a gadget that can be understood as a cut down version of McBride's one hole contexts [8]. Instead of recording the full path followed by our programs in the finite state machine describing the protocol, we merely record a de-looped version.

**Types of the Primitives** We include code snippets showing the types of the primitives dealing with communications. Note that the Idris 2's funny arrow (-0) denotes a linear function space, and consequently all of these primitives are linear in the channel they take as an input (ensuring it cannot be reused after the protocol has been stepped) and systematically return, wrapped in a linear IO monad called IO1, a channel indexed by the updated protocol. First, send and recv, the primitives dealing with sending arbitrary values back and forth. send takes an argument of type ty while recv returns one as the Result of the communication.

```
send : Channel (Send ty s) e -@ (ty -> IO1 (Channel s e))
recv : Channel (Recv ty s) e -@ IO1 (Res ty (\ _ => Channel s e))
```

Second, offer and select, the primitives dealing with branching. Both involve a choice in the form of a Bool named b that is used to compute the next channel state. offer gets it from the other thread as a Result of some communication while select requires the caller to pass it and will forward the decision to the other party.

```
offer : Channel (Offer s1 s2) e -@
IO1 (Res Bool (\ b => ifThenElse b (Channel s1 e) (Channel s2 e)))
select : Channel (Select s1 s2) e -@
((b : Bool) -> IO1 (ifThenElse b (Channel s1 e) (Channel s2 e)))
```

Third, unroll and roll, the primitives dealing with the iso-recursive nature of our (co)inductive protocols. The type of these primitives reveal the role of Channel's second parameter: it is

a stack of open session types corresponding to the bodies of the nested fixpoints encountered during execution. unroll opens a fixpoint and thus pushes a new entry onto the stack e while roll does the opposite.

```
unroll : Channel (Fix nm s) e -@ IO1 (Channel s (s :: e))
roll : Channel s (s :: e) -@ IO1 (Channel (Fix nm s) e)
```

Last but not least, **rec** steps a channel that has reached a recursive call to a fixpoint by looking up the open session and environment associated to the specified position in the stack and reinstating them.

```
rec : {pos : _} -> Channel (Rec pos) e -@
IO1 (Channel (Fix nm (SessionAt pos e)) (EnvAt pos e))
```

**Example Server** Omitting the somewhat horrifying types, we include the implementation of the server repeatedly offering to greet the user and execute the adder protocol that we sketched earlier. We use Idris 2's named argument syntax to explicitly pass the name nm to the unroll and rec calls in order to clarify what step of the protocol is being reached.

```
server id ch = do
                                         adder acc ch = do
 ch <- unroll {nm = Nu "ready"} ch
                                           ch <- unroll {nm = Nu "adder"} ch
 b # ch <- offer ch
                                           b # ch <- offer ch</pre>
 if b then close ch else do
                                           if b then do
    let msg = "Hello n°\{show id}!"
                                                ch <- send ch acc
                                                rec {nm = Nu "ready"} ch
    ch <- send ch msg
    ch <- adder 0 ch
                                              else do
    server (1 + id) ch
                                                (n # ch) <- recv ch
                                                ch <- rec {nm = Nu "adder"} ch
                                                adder (acc + n) ch
```

On the left-hand side we are acting as a **server** parametrised by a client id number. We first **unroll** the largest fixpoint corresponding to the 'ready' state repeatedly offering the adder protocol, and wait for the client to choose one of the branches we **offer**. If they are done we **close** the channel and terminate, otherwise we **send** a greeting, call the **adder** routine, and eventually continue acting as the server with an incremented id number.

One the right-hand side we are acting as the adder parametrised by an accumulator acc. We unroll the largest fixpoint corresponding to the 'adder' protocol and offer the client a choice between getting the running total or sending us more numbers. If they are done, we send back the value in the accumulator and, having reached a Rec variable in the protocol, use rec to jump back to the 'ready' state. Otherwise we recv an additional number, add it to the accumulator, use rec to jump back to the 'adder' state and continue acting as the adder.

### Limitations and Future Work

**Encoding Uniqueness via Linear Types** Unlike Brady's prior work in Idris 1 [2], we are using Idris 2 which does not currently have uniqueness types. We are forced to use the linearity granted to us by Quantitative Type Theory (QTT) [9, 1] to emulate uniquess via a library-wide invariant. This adds a degree of noise and trust to the implementation. We would ideally want a system combining both linearity and uniqueness to benefit from enhanced expressivity and efficiency as described by Marshall, Vollmer, and Orchard [7].

**Small Runtime Overhead** In the current version of our library, we compute a handful of key offset values by induction over the protocol. Correspondingly the protocol is not marked as erased anymore and, if compilation does not specialise the relevant combinators agressively, then we may very well be evaluating these relatively small computations at runtime. As far as we understand, directly adding typed staging to Idris 2 through the use of a two-level type theory à la Kovács [6] would not be enough to solve our issue: for typing purposes, the protocol does need to persist through staging. But it should be possible to move it from QTT's unrestricted to its erased modality.

**Ergonomics** We found writing the types of inner loops of programs with non-trivial communication patterns to be tedious as they expose quite a lot of the powerful but noisy encoding of syntaxes with binding we use. Figuring out a more user-friendly surface syntax for our protocols is left as future work.

**Totality Checking** Our lightweight embedding does distinguish smallest and largest fixpoints but Idris 2's productivity checker does not currently take advantage of the knowledge that the user is making steady progress in a potentially infinitely repeating protocol ( $\nu x$ . P) to see that a program is total. Experiments suggest we may be able to introduce a lightweight encoding of a Nakano-style modality [10] for guarded programming to let the user justify that their co-recursive calls are all justified by regular progress.

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